7.11 Quadric Surfaces

Point coordinates of the quadric surfaces: x, y, z
Real numbers: A, B, C, a, b, c, $k_1, k_2, k_3, \ldots$

699. General Quadratic Equation

$$Ax^2 + By^2 + Cz^2 + 2Fyz + 2Gzx + 2Hxy + 2Px + 2Qy + 2Rz + D = 0$$

700. Classification of Quadric Surfaces

Case	Rank(e)	Rank(E)	Δ	k signs	Type of Surface
1	3	4	<0	Same	Real Ellipsoid
2	3	4	>0	Same	Imaginary Ellipsoid
3	3	4	>0	Different	Hyperboloid of 1 Sheet
4	3	4	<0	Different	Hyperboloid of 2 Sheets
5	3	3		Different	Real Quadric Cone
6	3	3		Same	Imaginary Quadric Cone
7	2	4	<0	Same	Elliptic Paraboloid
8	2	4	>0	Different	Hyperbolic Paraboloid
9	2	3		Same	Real Elliptic Cylinder
10	2	3		Same	Imaginary Elliptic Cylinder
11	2	3		Different	Hyperbolic Cylinder
12	2	2		Different	Real Intersecting Planes
13	2	2		Same	Imaginary Intersecting Planes
14	1	3			Parabolic Cylinder
15	1	2			Real Parallel Planes
16	1	2			Imaginary Parallel Planes
17	1	1			Coincident Planes

Here

$$e = \begin{pmatrix} A & H & G \\ H & B & F \\ G & F & C \end{pmatrix}, \quad E = \begin{pmatrix} A & H & Q & P \\ H & B & F & Q \\ G & F & C & R \\ P & Q & R & D \end{pmatrix}, \quad \Delta = \det(E),$$

k_1, k_2, k_3 are the roots of the equation,

$$\begin{vmatrix} A-x & H & G \\ H & B-x & F \\ G & F & C-x \end{vmatrix} = 0.$$

701. Real Ellipsoid (Case 1)

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

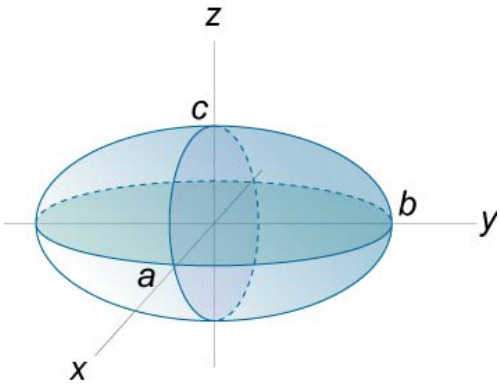

Figure 142.

702. Imaginary Ellipsoid (Case 2)

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=-1$$

703. Hyperboloid of 1 Sheet (Case 3)

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$$

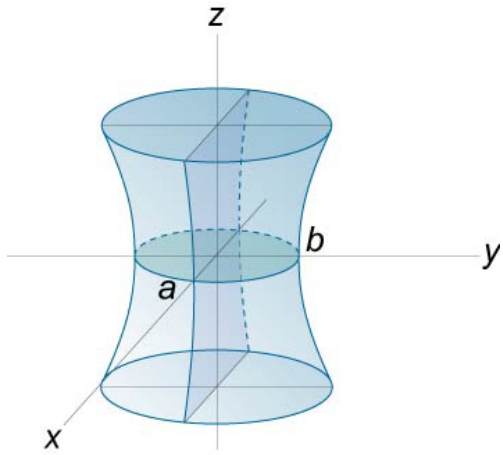

Figure 143.

704. Hyperboloid of 2 Sheets (Case 4)

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1$$

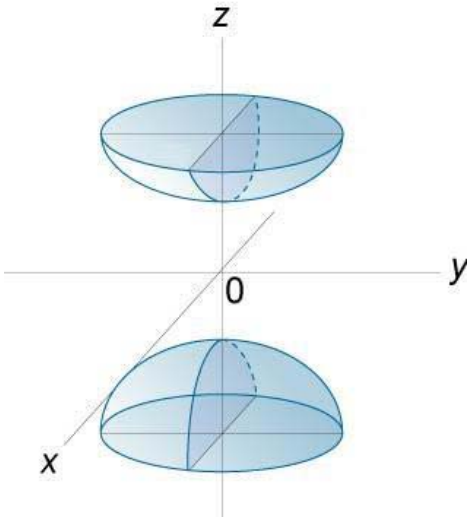

Figure 144.

705. Real Quadric Cone (Case 5)

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$$

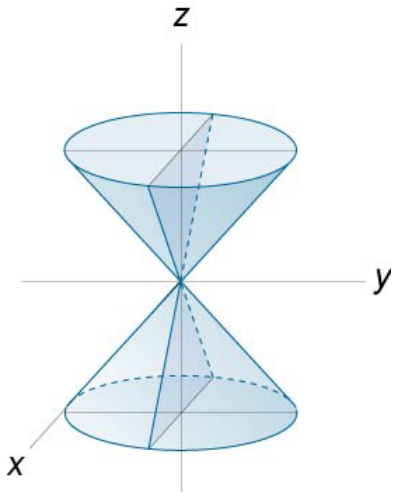

Figure 145.

706. Imaginary Quadric Cone (Case 6)

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=0$$

707. Elliptic Paraboloid (Case 7)

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-z=0$$

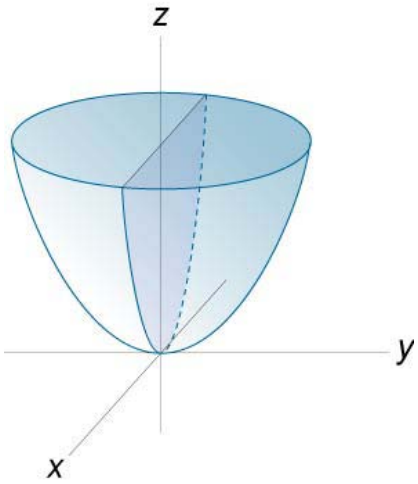

Figure 146.

708. Hyperbolic Paraboloid (Case 8)

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} - z = 0$$

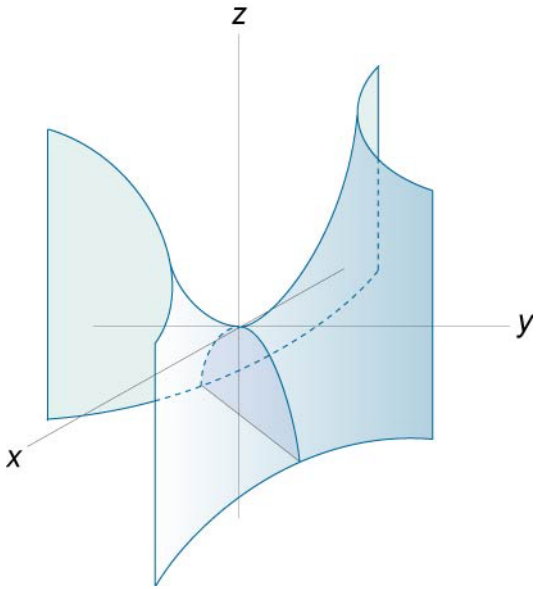

Figure 147.

709. Real Elliptic Cylinder (Case 9)

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

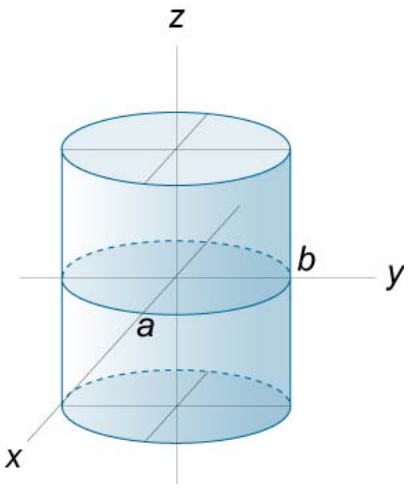

Figure 148.

710. Imaginary Elliptic Cylinder (Case 10)

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=-1$$

711. Hyperbolic Cylinder (Case 11)

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

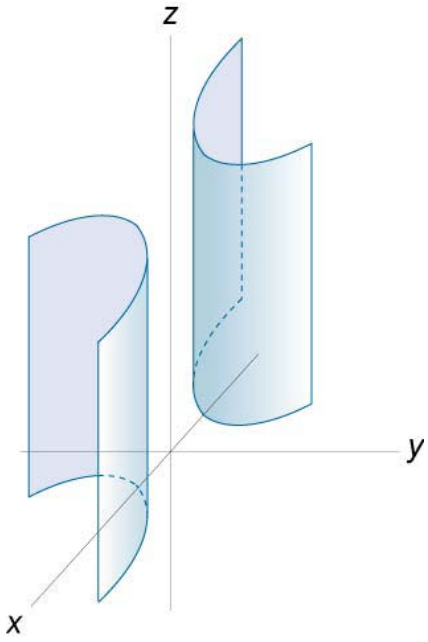

Figure 149.

712. Real Intersecting Planes (Case 12)

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$$

713. Imaginary Intersecting Planes (Case 13)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 0$$

714. Parabolic Cylinder (Case 14)

$$\frac{x^2}{a^2} - y = 0$$

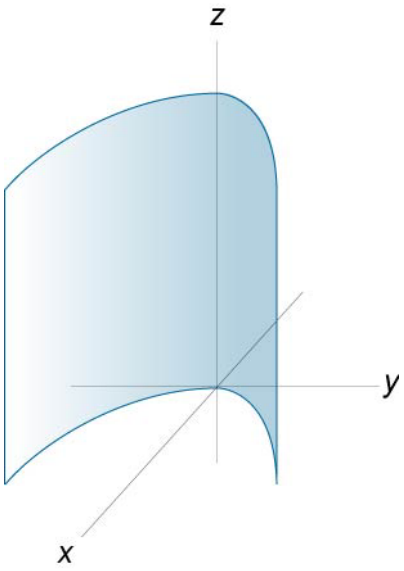

Figure 150.

715. Real Parallel Planes (Case 15)

$$\frac{x^2}{a^2} = 1$$

716. Imaginary Parallel Planes (Case 16)

$$\frac{x^2}{a^2} = -1$$

717. Coincident Planes (Case 17)

$$x^2 = 0$$